

## Where did I go wrong?

Puzzlement in front of a multi-solution Su Doku puzzle.

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I want to relate my experience trying to solve a puzzle that I thought I had transcribed from of a book I bough recently<sup>1</sup> because I was mystified by my first result before I understood the vastness of the problem. But before going further I want to say that I made a mistake in transcribing it, forgetting one of the clues (35.5 in red in the Fig. 1). But I did not find that mistake by myself; it was pointed to me by the author of SudokuJM<sup>2</sup> when he was trying to reproduce with his program some of my manual output. I have reworked some parts of this article to take that error in account but I have tried to preserve my original feelings when I first dealt with this stepped puzzle.

Let us first establish our basis: the puzzle and its “solution” directly from the book.

	1	4							8	9	1	4	3	6	2	5	7
			7	8	6		1		5	2	4	9	7	8	6	3	1
			5		9				6	3	7	2	5	1	9	8	4
8						2	3		4	8	6	7	9	5	1	2	3
1	3				5	6			7	1	3	8	2	4	5	6	9
9	5					7			9	5	2	6	1	3	4	7	8
	5		4						2	7	5	3	4	9	8	1	6
3	9	1	8						3	6	9	1	8	2	7	4	5
				7	3				1	4	8	5	6	7	3	9	2

Fig. 1 Original puzzle #88 (degrees of freedom 162) and proposed solution

From now on, I will refer to that puzzle (without the “red” 5) as P-88 to differentiate it from the original G-88. Let us at that occasion note that any solvable puzzle can remain solvable when one clue is removed; no basic rule is contravened by a removal, but as less information is available (fewer clues) player’s intervention by the addition of a chosen value may become necessary. I have used the “A template for helping with Su Doku puzzles”<sup>3</sup> in my attempts at solving this puzzle. With the terminology I proposed in that article, this puzzle is a

<sup>1</sup> “100 Su Doku no. 1” by Wayne Gould, (Édition Générales First, Paris) originally published as “The Times Su Doku Book 1” (Harper Collins Publishers)

<sup>2</sup> SudokuJM.exe is a stand alone Sudoku grid creator and solver. It can be obtained from [www.sudokujm.com](http://www.sudokujm.com)

<sup>3</sup> See my site [www.paris-pc-gis.com/LUDIK/varia\\_main.htm](http://www.paris-pc-gis.com/LUDIK/varia_main.htm) under the Sudoku heading

“stepped” one; the application of automatic rules (assignments of integers to specific locations – a stage that some refer to as “deterministic” – ) is followed by a choice made among the integers that can be assigned to an open position. When a choice is entered, it may be followed by another automatic assignment stage; the final outcome of a choice plus automatic assignment is one of three possibilities: the puzzle is solved (bravo!), another choice is required (another step must be undertaken) or the choice results in some unacceptable assignment (that step must be redone from its start with an acceptable alternate integer to the one chosen originally).

The procedure I followed is quite simple. I used first the template for the original automatic assignments and I got the following puzzle (Fig. 2).

	1	4			
	4		7 8	6	1
	7			9	
4	8	6	7	1	2 3
7	1	3	8	2	4 5 6 9
9	5	2			7
	5		4		
3		9	1	8	
	8			7	3

Fig. 2 P-88 after automatic assignments  
(dof=101)

The assigned integers are exactly those of the proposed solution, except the missing 5 in 35. To start the next step, one value must be specified. I prefer choosing from a cell that could be filled by one of two integers. It is essentially a question of efficiency; if a cell is chosen from the “global performance table” of the template with the value 2, it means that if one of the two acceptable digits does not work the other must. But nothing prevents the solver from choosing any among the acceptable more than 2 values in a position.

To be completely methodical, one should try all the possibilities offered at a given step. That would require accumulating a large number of alternatives to be multiplied at each new step until all the available memory is used up. There is however a welcome relief because alternative choices can lead to identical outcomes, reducing thus the number of possible alternatives that should be considered as a basis for the next step.

I have adopted a faster procedure, in line with my proposal for measuring a puzzle difficulty. Among the possible alternatives for a given step, I choose the one that yields the smaller number of “degree of freedom” (calculated as the sum of the entries of the global performance table minus the number of non-null entries).

4	4	0	0	4	5	3	3	4											
2	3	0	4	0	0	0	2	0		1	4			8			6	3	1
4	3	0	4	4	5	0	4	4		4	7	8		9	4				
0	0	0	0	2	2	0	0	0		8	7								
0	0	0	0	0	0	0	0	0		4	8	6	7			1	2	3	
0	0	0	2	3	3	2	0	2		7	1	3	8	2	4	5	6	9	
3	3	0	4	0	4	3	3	4		9	5	2				8	7	4	
0	4	0	0	0	3	3	2	5											
3	3	0	4	3	0	0	4	4											
										5			4						8
										3	9	1	8			4	5		
										4	8			7	3				

Fig. 2 Start of the next step  
 Global performance table after auto- After choosing 8 in r6c7 and applying automatic assignments. All alternatives automatic rules (dof=65) for "binary" cells were considered.

Two more steps are required for reaching a solution

5	1	4	9	8															
2	9	4	5	7	8	6	3	1											
8	7					9	4	5											
4	8	6	7	9	5	1	2	3											
7	1	3	8	2	4	5	6	9											
9	5	2				8	7	4											
		5		4				8											
3		9	1	8		4	5												
	4	8		5	7	3													
5	1	4	9	8															
2	9	4	5	7	8	6	3	1											
8	7			1	2	9	4	5											
4	8	6	7	9	5	1	2	3											
7	1	3	8	2	4	5	6	9											
9	5	2			1	8	7	4											
6	7	5	9	4	3	2	1	8											
3	2	9	1	8	6	4	5	7											
1	4	8	2	5	7	3	9	6											
5	3	1	4	6	9	7	8	2											
2	9	4	5	7	8	6	3	1											
8	6	7	3	1	2	9	4	5											
4	8	6	7	9	5	1	2	3											
7	1	3	8	2	4	5	6	9											
9	5	2	6	3	1	8	7	4											
6	7	5	9	4	3	2	1	8											
3	2	9	1	8	6	4	5	7											
1	4	8	2	5	7	3	9	6											

Fig. 3 Two more steps and the solution.  
 2 in r2c1 (dof=37)                      7 in r1c7 (dof=6)                      3 in r1c2

The "solution" is not, and by far, that of the book. Where did I go wrong? Did I make a mistake? I was really puzzled.

I verified immediately that the solution was acceptable, and it is. Does the puzzle offer multiple solutions, and how many? I had heard of such puzzles but they were presented as oddities; I was also convinced that commercially available Su Duko puzzles will offer only one solution, and here was one with unmistakably "multiple" solutions, offering at least 2. Then I became aware that the last choice (3 in r1c2) had an alternate I had not tried at first and there it worked too. (Fig. 4)

5	6	1	4	3	9	7	8	2
2	9	4	5	7	8	6	3	1
8	3	7	6	1	2	9	4	5
4	8	6	7	9	5	1	2	3
7	1	3	8	2	4	5	6	9
9	5	2	3	6	1	8	7	4
6	7	5	9	4	3	2	1	8
3	2	9	1	8	6	4	5	7
1	4	8	2	5	7	3	9	6

Fig. 4 6 in r1c2 gives another different but valid solution

Was that all? I started to feel a little weird before beginning again a little more systematically and very quickly I accumulated 6 new ones

5	3	1	4	6	9	2	8	7
2	9	4	5	7	8	6	3	1
8	6	7	2	3	1	9	4	5
4	8	6	7	9	5	1	2	3
7	1	3	8	2	4	5	6	9
9	5	2	3	1	6	8	7	4
1	2	5	6	4	3	7	9	8
3	7	9	1	8	2	4	5	6
6	4	8	9	5	7	3	1	2

5	3	1	4	6	9	2	8	7
2	9	4	5	7	8	6	3	1
8	6	7	2	3	1	9	4	5
4	8	6	7	9	5	1	2	3
7	1	3	8	2	4	5	6	9
9	5	2	6	1	6	8	7	4
1	2	5	3	4	6	7	9	8
3	7	9	1	8	2	4	5	6
6	4	8	9	5	7	3	1	2

5	3	1	4	6	9	2	8	7
2	9	4	5	7	8	6	3	1
8	6	7	2	1	3	9	4	5
4	8	6	7	9	5	1	2	3
7	1	3	8	2	4	5	6	9
9	5	2	6	3	1	8	7	4
1	2	5	3	4	6	7	9	8
3	7	9	1	8	2	4	5	6
6	4	8	9	5	7	3	1	2

  

5	6	1	4	3	9	2	8	7
2	9	4	5	7	8	6	3	1
8	3	7	2	1	6	9	4	5
4	8	6	7	9	5	1	2	3
7	1	3	8	2	4	5	6	9
9	5	2	6	6	1	8	7	4
1	2	5	6	4	6	7	9	8
3	7	9	1	8	2	4	5	6
6	4	8	9	5	7	3	1	2

5	6	1	4	3	9	2	8	7
2	9	4	5	7	8	6	3	1
8	3	7	2	6	1	9	4	5
4	8	6	7	9	5	1	2	3
7	1	3	8	2	4	5	6	9
9	5	2	3	1	6	8	7	4
1	2	5	6	4	3	7	9	8
3	7	9	1	8	2	4	5	6
6	4	8	9	5	7	3	1	2

5	6	1	4	3	9	2	8	7
2	9	4	5	7	8	6	3	1
8	3	7	2	6	1	9	4	5
4	8	6	7	9	5	1	2	3
7	1	3	8	2	4	5	6	9
9	5	2	6	1	3	8	7	4
1	2	5	3	4	6	7	9	8
3	7	9	1	8	2	4	5	6
6	4	8	9	5	7	3	1	2

Fig. 5 6 other solutions

But none resembles the proposed solution. A major difference is the upper left cell (8 in proposed solution) for which 4 different integers are acceptable (see the first "global performance table") and that could not be reached early the way I worked. I imagined then to "force" some choices after the first automatic assignment and this is what happened.

	1	4						
	4		7	8	6		1	
	7				9		4	
4	8	6	7			1	2	3
7	1	3	8	2	4	5	6	9
9	5	2				4	7	8
		5		4				
3		9	1	8				4
	4	8			7	3		

Fig. 6a After original auto assignments (Fig. 2 dof=110)

Assigning

8 to r1c1  
9 to r1c2  
5 to c2r1

8	9	1	4			5		
5	2	4	9	7	8	6	3	1
6	3	7				9	8	4
4	8	6	7			1	2	3
7	1	3	8	2	4	5	6	9
9	5	2				4	7	8
		5		4		8		
3		9	1	8				4
	4	8			7	3		

Fig. 6c After auto assignments (dof=41)

8	9	1	4	6	3		5	
5	2	4	9	7	8	6	3	1
6	3	7		1		9	8	4
4	8	6	7			1	2	3
7	1	3	8	2	4	5	6	9
9	5	2	6	3	1	4	7	8
		5	3	4		8		
3		9	1	8			4	5
	4	8			7	3		6

Fig. 6d 6 in r1c5 (dof =19)

8	9	1	4	6	3		5	
5	2	4	9	7	8	6	3	1
6	3	7	2	1	5	9	8	4
4	8	6	7	5	9	1	2	3
7	1	3	8	2	4	5	6	9
9	5	2	6	3	1	4	7	8
1		5	3	4		8	9	
3		9	1	8			4	5
2	4	8	5	9	7	3	1	6

Fig. 6e 2 in r3c4 (dof=8)

8	9	1	4	6	3	2	5	7
5	2	4	9	7	8	6	3	1
6	3	7	2	1	5	9	8	4
4	8	6	7	5	9	1	2	3
7	1	3	8	2	4	5	6	9
9	5	2	6	3	1	4	7	8
1	7	5	3	4	6	8	9	2
3	6	9	1	8	2	7	4	5
2	4	8	5	9	7	3	1	6

Fig 6f1 2 in r1c7

8	9	1	4	6	3	7	5	2
5	2	4	9	7	8	6	3	1
6	3	7	2	1	5	9	8	4
4	8	6	7	5	9	1	2	3
7	1	3	8	2	4	5	6	9
9	5	2	6	3	1	4	7	8
1	6	5	3	4	2	8	9	7
3	7	9	1	8	6	2	4	5
2	4	8	5	9	7	3	1	6

Fig. 6f2 7 in r1c7

These dual solutions resemble more the proposed one because of the 3 integers that have been "forced" but they are still different. This is a simple proof that this puzzle offers a large number of solutions, 10 plus the proposed one are already identified here and it is just a superficial scratch, much too many to reassure a Su Doku'ist in knowing that the proposed solution is THE solution.<sup>4</sup>

<sup>4</sup> I had the chance since then to run a beta advanced version of SudokuJM with that P-88 puzzle and it churned vigorously for quite a time to identify 151 different solutions.

Once I recognized my early mistake, I realized that the procedures I used and the findings are made remained valid; that early adventure in the realm of multiple solution puzzles has been instrumental in establishing a solid experimental basis for my search in detecting and solving multiple solution puzzles.

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